Participation of Wind Power in Electricity Markets.

Julio Usaola
Departamento de Ingeniería Eléctrica
Universidad Carlos III de Madrid
e-mail: jusaola@ing.uc3m.es

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Contents

1 Introduction. **2**

2 Wind Power and Electricity Markets. **2**
2.1 Imbalance Costs. 3
2.2 Revenues of a Market Participant. 4

3 Short term wind power prediction. Uncertainty. **6**
3.1 Short term wind power prediction. 6
3.2 Uncertainty of short term wind power prediction. 7

4 Evaluation of Imbalance Costs for Wind Producers. **9**

5 Optimal bids. **10**

6 Dependence of market prices on the wind power submitted. **11**

7 Study Cases. **12**
7.1 Uncertainty of Wind Power Predictions. 12
7.1.1 Assumptions. 12
7.1.2 Data. 13
7.1.3 Results and Comments. 13
7.2 Uncertainty of Wind Power Predictions and Intraday Prices. 15
7.2.1 Price predictions. 15
7.2.2 Results and Comments. 16

8 Conclusion. **17**

9 Future studies. **17**

A Statistical distributions. **18**
A.1 Beta distribution. 18
A.2 Binomial distribution. 18
A.3 Exponential distribution. 18

1 Introduction.

Nowadays, wind power is steadily increasing in the power networks, and its participation in the electricity markets is also growing. Since it is a source of power not much dependable, and with limited predictability, the imbalance costs will be likely high. Participation in the electricity markets may prevent cross-subsidies for the incurred imbalance costs.

The imbalance market, is, therefore, a keystone for the integration of wind power into electricity markets, and its rules affect the wind producer strategies and revenues. The losses due to imbalance costs have been evaluated by different authors and they coincide roughly in their conclusions.

Participation of wind energy in electricity markets requires an estimate of the future production that is made by a forecasting tool. These tools have evolved rapidly in the last years but they still have a limited accuracy, and progress in this field will be likely slow. But these tools may have sophisticated outputs such as the statistical modelling of the uncertainty.

The possibility of knowing this uncertainty allows wind power participants in markets to optimize their position in order to maximize their revenue. This optimization is based on arbitraging between the market and the imbalance price.

The studies performed up to now, however, are a rough simplification of reality. The uncertainty modelling is being developed now, specially the spacial and temporal dependence between forecasts. And the impact of such an arbitraging on markets with limited liquidity is still to be evaluated.

The present report intends to highlight some of these issues, giving a summary of recent studies, and reporting new simulation studies that gives an insight into some not yet studied subjects. The report is structured as follows. Firstly, a short introduction to electricity markets is given, focusing on those aspects more relevant to wind power participation. Then a survey of studies quantiying the imbalance costs for wind producer is given. The formulation of the optimization of bids in an intraday market follows, and then, the results of certain simulation studies are given. A Conclusion section ends the report. Appendices with statistical information are also given.

2 Wind Power and Electricity Markets.

Most of electricity markets are organized in a sequential way, due to the need of fitting exactly production and demand at each time. The sequence begins with the Forward Markets (FM), with times ranging months to the day before the Operational Settlement Period (OSP). Daily Markets (DM)follow, with commitments being made (typically) between 14 and 38 hours before the OSP. The Intraday Markets (IM) have shorter time scheduling and they overlaps with the Balancing markets (BM) where the imbalances between scheduled generation and actual demand are finally settled.

When wind energy may participate in the electricity markets, it must interact in such a scheme. Since wind energy cannot be programmed in advance, it is necessary to have a prediction of the production for the next OSP, and this prediction is available with a rather acceptable reliability 24-48 hours before. Therefore, wind energy participates in the DM, IM and BM. Normally, wind power producers participate in DM, committing themselves for the power predicted before the next gate closure. This means usually to make a bid for the DM and to update it in the ID markets, when predictions with lower errors are available. In the BM, the imbalances are settled and paid at the imbalance cost. Since the accuracy of wind power predictions is not very high, imbalance markets are very important for wind power producers, and their rules affects them especially.

Some problems may present with the participation of large amounts of wind power in the ID markets. Firstly, the ID markets have less liquidity than the DM, and the impact of large amounts of wind power on them (very likely buying or selling energy at the same time together) may be significant. Besides, if the gate closure is very close (30-60 minutes) before the OSP, the updating of the previous positions may be more accurate, avoiding too much back and forth negociation of energy. But on the other hand, more frequent IM have less liquidity, and the reserve management may not be optimal with such short time scheduling. An optimal time horizon in these systems with high penetration must be compromised.

2.1 Imbalance Costs.

Principally, there are two types of imbalance price mechanisms[1]:

- Dual imbalance pricing, where a different price is applied to positive imbalance volumes and negative imbalance volumes (two prices system); or
- Single imbalance pricing, where a single imbalance price is used for all imbalance volumes (one price system).

Each of these mechanisms is used in a number of European markets. Of relevance to the reference model is the case of the Nordel market, where dual imbalance pricing is used in Sweden, Finland and Denmark, and single imbalance pricing in Norway. This shows that a regional market can function even if constituent areas have different imbalance pricing principles. However, it may be the case that efficiency would be enhanced by the application of a common methodology and recommended.

Where a dual imbalance pricing regime is employed, it is the "main" price that is derived from energy balancing actions. The "reverse price" can be determined by reference to a power exchange (e.g. Elspot in the Nordel market, APX in Great Britain and Powernext in France) or be based on the prices of the balancing actions in the reverse direction during the settlement period. (The main price is that applied to imbalance volumes in the same direction as the overall market, where as the reverse price is that applied to imbalance volumes opposite in direction to the overall market e.g. "short" when the market is "long", or vice versa). Dual imbalance can be incentive for BRP[2] to manage their position in a more secure way for the system.

There are also two principal methods by which these imbalance prices can be determined:

- Average price of energy balancing actions; or
- Marginal price of energy balancing actions.

A third methodology - the average price of the marginal 500MWh of energy balancing actions - has been introduced in Great Britain in November 2006.

Imbalance prices calculated on a marginal basis will tend to over-recover when compared to expenditure on balancing actions, but only in a two price system and there are mitigation measures for this. In some countries, such as Sweden, this profit is retained by the TSO, where as in e.g. Denmark, the Netherlands and France the profit is socialised or redistributed to parties. Finally, in other countries, such as Great Britain, the entire imbalance charging receipts are redistributed to parties (and the costs of balancing actions are recovered through Use of System charges).

Clearly, there are a number of different imbalance pricing regimes that can be generated using the above principles. There will be advantages and disadvantages of each, and the example of Nordel has shown that it is not necessary to have a common mechanism over an integrated region. However, in developing this reference model ETSO TF BM recommends that a harmonised imbalance pricing methodology be implemented. This would provide common signals to balance and consistency in the recovery of costs, and would therefore maximise efficiency. As imbalance risks (expected imbalance price $\times$ imbalance volumes) will be accounted for by the market parties in their energy prices, harmonised imbalance definitions and pricing methodologies would also make market prices more comparable, again improving efficiency of trade across balancing regimes.

The two price system scheme used in the Spanish electricity market may be represented in Figure 1. In this figure, MP means Marginal Price, BP Buy Price, and SP Sell Price.

In Figures 2 to 6 the average monthly costs of imbalance price relative to marginal market price are represented.

[1] This section has been taken mostly from [ETSO 2007].

[2] Balance Responsible Party, who is the responsible for keeping the net balance on all the connections within its control and faces the liability consequences if this is not achieved. The liability in case of imbalance involves the payment of an imbalance charge to the operator of the market area who is responsible for keeping the balance in the area.

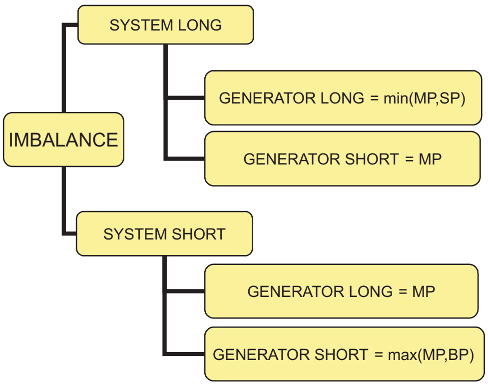

Figure 1: Imbalance cost in the Spanish system.

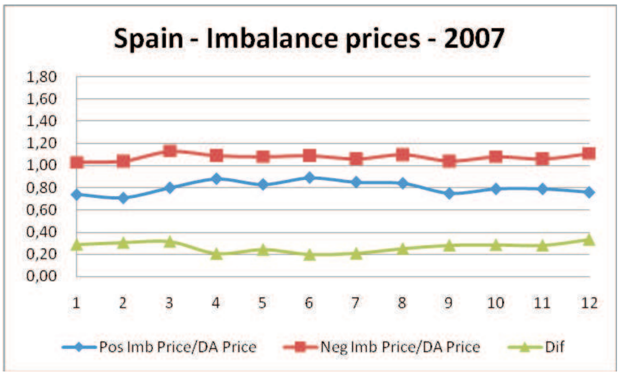

Figure 2: Average monthly imbalance cost in the Spanish system in 2007.

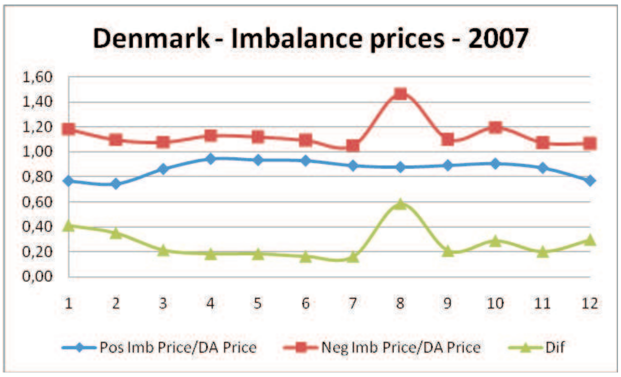

Figure 3: Average monthly imbalance cost in the Danish system in 2007.

2.2 Revenues of a Market Participant.

In the approach followed here, the time scheduling of the Spanish Electricity Market will be followed loosely. It will be considered a DM with gate closure at 11 AM the day before the OSP, and six ID markets equally espaced among the day, with three hours before the beginning of the OSP.

Hence, in general terms, the revenue R for a given wind farm in a pool market may be generalized as:

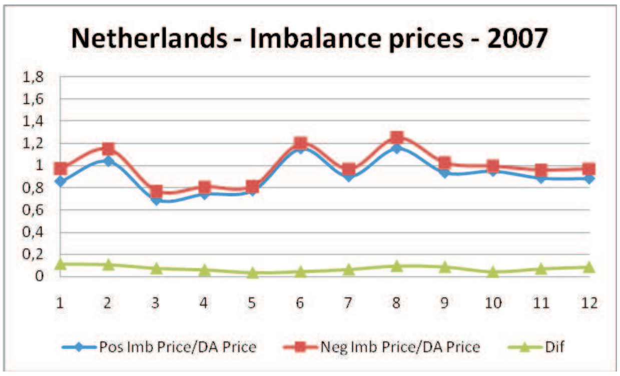

Figure 4: Average monthly imbalance cost in the Dutch system in 2007.

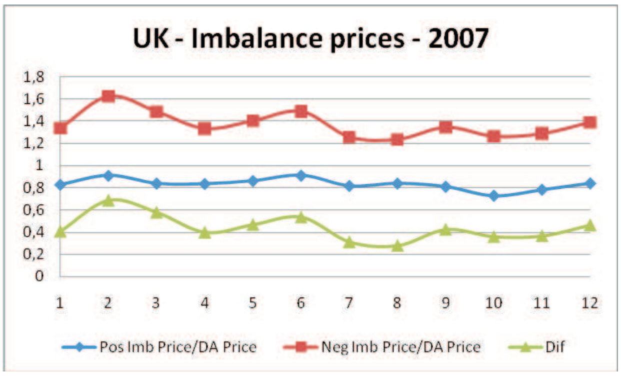

Figure 5: Average monthly imbalance cost in the UK system in 2007.

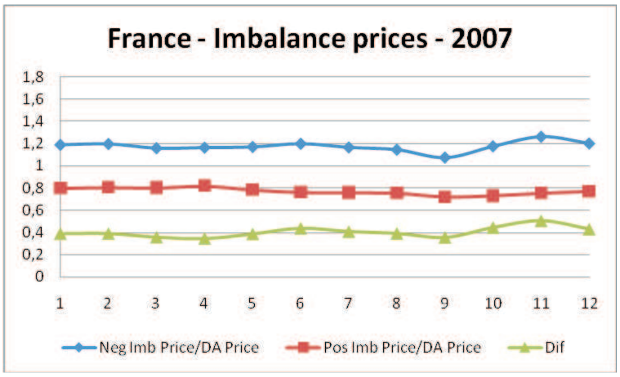

Figure 6: Average monthly imbalance cost in the French system in 2007.

$$R = \sum_{t=1}^{T} [P_{d,t}\pi_{d,t} + \pi_{i,t}(P_{i,t} - P_{d,t}) + IC_t] \tag{1}$$

Where IC is the imbalance cost, that have the general expression

$$IC_t = \begin{cases} \pi_t^{sell}(P_{g,t} - P_{i,t}) & P_{g,t} > P_{i,t} \\ \pi_t^{buy}(P_{g,t} - P_{i,t}) & P_{g,t} < P_{i,t} \end{cases} \quad (2)$$

The meaning of the different terms of the equations are:

$P_{g,t}$	Power actually generated by the wind farm in the hour t.
$P_{d,t}$	Power committed to the wind farm in the daily market for the hour t. It coincides with the prediction available at the gate closure of the daily market.
$P_{i,t}$	Power committed to the wind farm in the intraday market for the hour t. It coincides with the prediction available at the gate closure of the intraday market.
$\pi_{d,t}$	Marginal price of energy in the daily market for the hour t.
$\pi_{i,t}$	Marginal price of energy in the intraday market for the hour t.
π_t^{sell}	Imbalance price for spill energy at hour t.
π_t^{buy}	Imbalance price for energy bought at hour t.

In the paper, the time sequence of the different markets are shown in Figure 7, where P is the moment when the predictions are produced. In this figure, it is assumed that there is a daily market and six intraday markets that take place each four hours. It can be seen that the bids should be presented 14 hours before the beginning of the daily market, and 3 hours before the beginning of the intraday markets. This means that the predictions for the daily markets must have a time horizon between 14 and 367 hours, while the predictions for the intraday markets are produced between 3 and 6 hours before the operation time. The updating, then, means that the updated predictions are between 10 and 30 hours after the daily market prediction.

Hours	11	12	13	14	15	16	17	18	19	20	21	22	23	24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Daily	P														D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
ID1											P				ID1	ID1	ID1	ID1																				
ID2																P			ID2	ID2	ID2	ID2																
ID3																				P			ID3	ID3	ID3	ID3												
ID4																								P			ID4	ID4	ID4	ID4								
ID5																												P			ID5	ID5	ID5	ID5				
ID6																																P			ID6	ID6	ID6	ID6

Figure 7: Time scheduling followed in the paper.

3 Short term wind power prediction. Uncertainty.

3.1 Short term wind power prediction.

Short term wind power prediction programs are tools that provide an estimation of the future power production of a wind farm, or a group of wind farms, in the next hours. For this purpose, they use meteorological forecasts coming from a Numerical Weather Prediction (NWP) tool, and sometimes real time SCADA data from the wind farms, as wind power production, measured wind speed, etc. Data of the wind farms, such as rated power, type and availability of wind turbines, etc. are also necessary. The output of these programs is the hourly average wind farm production for the next hours. Typically, predictions are issued for the next 48 hours, but longer time horizons are possible, sometimes at the price of a lower accuracy.

These predictions tools are less accurate than load prediction programs and their accuracy decreases with the time horizon. A survey of the accuracy of these tools is given in [Kariniotakis 2006], and an example for a typical wind farm, where the output from the prediction program SIPREOLICO is compared to persistence is shown in Figure 8. Persistence is a prediction method that assumes that the future production, for the entire time horizon considered, is the current production of the wind farm, i.e. $\widehat{p}(t+k|t) = p(t)$, where $\widehat{p}(t+k|t)$ is the power predicted at time t for k hours later, and $p(t)$ is the wind farm generated power at time t. This method is considered as a threshold of the performance of a forecasting method. SIPREOLICO is a prediction program used since year 2002 in Red Eléctrica de España, the Spanish TSO, and developed by Universidad Carlos III de Madrid, to forecast wind power production of the next 42 hours, every 15 minutes, for the 14 GW of wind power connected to the Spanish peninsular grid. Details of SIPREOLICO can be found in [Gonzalez 2004].

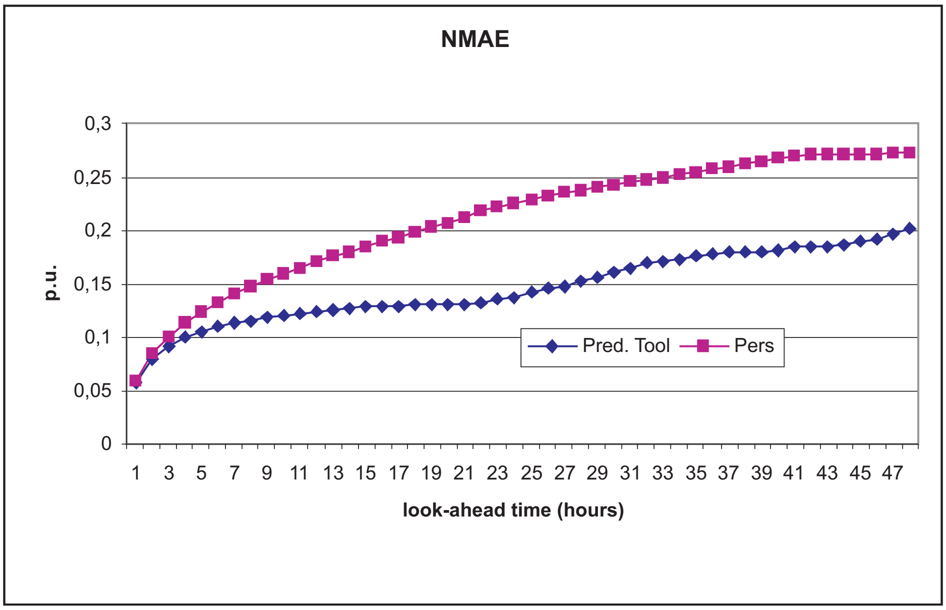

Figure 8: NMAE of SIPREOLICO and persistance for a typical wind farm.

Figure 8 represents the Normalized Mean Average Error, defined as

$$NMAE(k) = \frac{1}{P_n} \frac{\sum_{t=1}^{N} |e(t+k|t)|}{N} \tag{3}$$

Where

$$e(t+k|t) = p(t+k) - \widehat{p}(t+k|t)$$

And $p(t+k)$ is the production of the wind farm at time $(t+k)$. P_n is the nominal power of the wind farm, and N is the number of predictions examined along the considered time. It can be seen that the wind power prediction accuracy allows for much uncertainty, and that the actual value may differ widely from the predicted one.

3.2 Uncertainty of short term wind power prediction.

The predictions provided by a short term wind power prediction program are uncertain, and it is interesting to estimate this uncertainty in order to have more information about the future production of a wind farm. Let p be the random variable associated with the power output of a wind farm. Then, the probability of producing p MW, having predicted $\widehat{p}$ MW k hours before, is given by the probability density function $f_{\widehat{p},k}(p)$. The uncertainty, and hence the probability density function, changes with the range of the wind farm power otuput, since this value is bounded between zero and the rated power. Besides, the power curve of a wind turbine or wind farm is nonlinear. If we assume that the wind power predictions have gaussian uncertainty, then the probability density functions of the power predictions will not be gaussian. The shape of these probability density functions is also affected by the time lag elapsed between the prediction and the operation times. As shown before, predictions with a shorter time lag are more accurate, and the variance of their uncertainty distribution is likely to be smaller than those predictions produced longer before. To obtain analytically, or in real time, the uncertainty of this prediction is difficult, but accurate estimations can be made from past data, and some research has already been made in this field. Given the past predictions and wind production for these predictions, the accuracy of these predictions can be tabulated, and then their frequency can be used as an approximation of these probability density functions. If the power range of a wind farm is comprised between 0 and P_{max}, and this range is divided in Q intervals, the power p would be included in the interval q, if

$$\frac{q-1}{Q} P_{max} \leq p \leq \frac{q}{Q} P_{max}$$

The probability density function $f_{\widehat{p},k}(p)$ changes into $f_{\widehat{q},k}(p)$, where $\widehat{q}$ is the interval in which the predicted power $\widehat{p}$ is included. As an example, the following figures give the frequency distributions of the produced powers for different values and time lags of the prediction. Figure 9 shows frequency distributions when a low power had been predicted 7 hours before real time, for different power levels, while Figure 10 shows the frequency distribution for different power levels 36 hours before operating time. All these values have been obtained from real production of three months of a wind farm whose rated power has been normalized to 1.

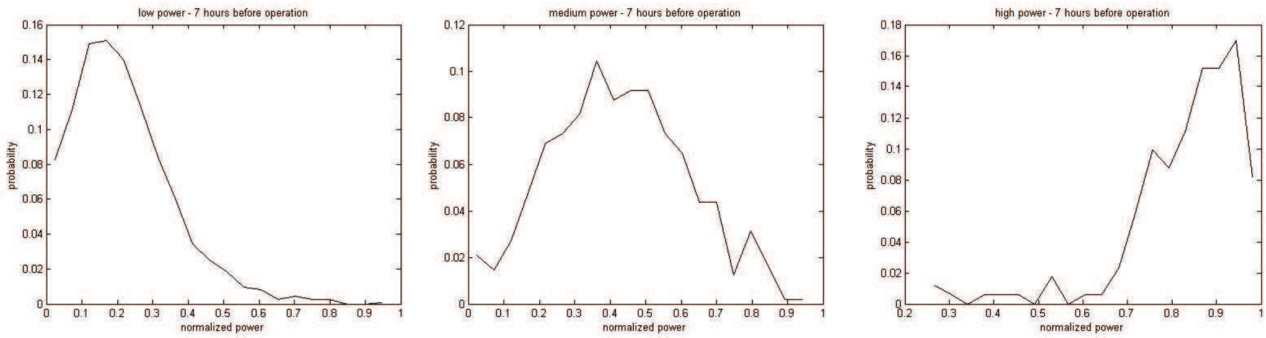

Figure 9: Probability density functions for $q^* = 2, 7, 13$ and $k = 7$ $Q = 14$.

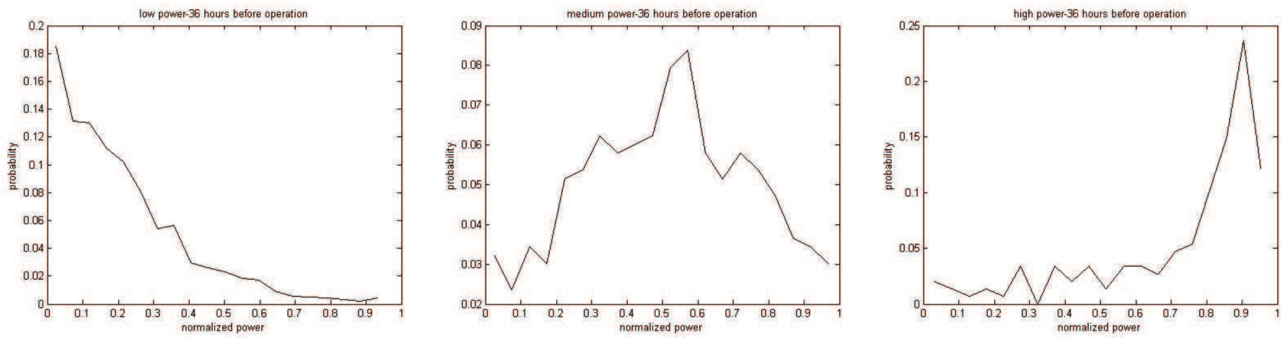

Figure 10: Probability density functions for $q^* = 2, 7, 13$ and $k = 36$ $Q = 14$.

It is not the purpose of this work to propose a model for this uncertainty, and a reasonable assumption will be used as an approximation. Due to the bounded nature of the power produced by a wind farm, a Beta PDF will be used, as proposed in [Fabbri 2005]. Heuristic PDF, as shown in [Usaola 2007], supports this assumption, although this is still an open field for research. The Appendix A.1 gives the analytical expression of Beta distribution. In our case, the mean of the distribution will be the predicted power at the time of interest, while the standard deviation σ will depend on the level of power injected, with respect to the wind farm rated power. This dependence has been obtained heuristically for some wind farms, and the results are shown in Figure 11, where the value of standard deviation is normalized to the rated power of the wind farm. Although there are wide variations, an approximation by a quadratic curve (shown in the picture) will provide realistic results.

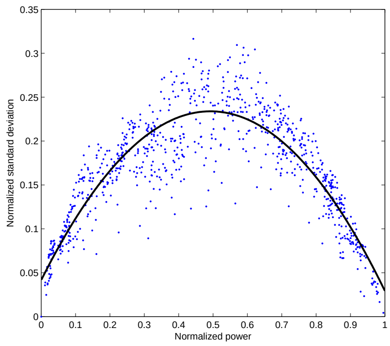

Figure 11: Relation between standard deviation and mean for the uncertainty of predictions.

The uncertainty of short term wind power prediction of geographically close wind farms are correlated, since the wind power in all of them are due to similar meteorological simulations. This dependence has not been modeled up to now, but the studies such as [Focken 2002] show the dependence between productions in a wide area (Germany). These results may be considered as an estimation of actual correlation values, although it is necessary to wait until more specific studies are made.

Another important dependence to be considered is time dependence between the uncertaitny of succesive predictions. This is necessary to be taken into account when a time sequence of predictions is used, as here.

4 Evaluation of Imbalance Costs for Wind Producers.

When wind power producers take part of an electricity market, they must behave as any other market participant (apart from the subsidies) and therefore they must commit themselves for a production level in the settlement period. Their production is estimated by means of a short term wind power prediction program that has a limited accuracy. This means that there will very likely be an imbalance between the forecasted and the actual production, which implies an imbalance cost, and hence economic losses.

It is interesting to quantify the importance of these losses. Their amount depends mainly on the accuracy of the prediction tool and the imbalance price. For the predictions' accuracy it can be said that they are quite similar for the existent prediction tools, since they depend mainly on the quality of the meteorological forecast that are at their base. For some of these tools, it also depends, to a lesser extent, on the time horizon of the prediction.

In general, it could be said that the economic losses linked to imbalance costs are about the 10% of the maximal revenue (i.e. the revenue that a farm would have if the power predictions were perfect) when participating in a daily market - typically between 13 and 37 hours in advance. This figure is an average obtained by numerous studies available in technical literature. For instance [Holttinen 2005] obtains losses of 12.11% in a simulation study that considers an aggregation of 2000 MW, following the Danish market rules. The Spanish market rules have been followed in similar studies, as in [Fabbri 2005], who obtains a figure of 11.5% for the losses under similar conditions. The results in [Usaola 2007] give a 10.39% losses and in [Angarita 2007(1)], a 9% of the maximum revenue is obtained. [Pinson 2007] accounts for a 13.1%, following the rules of the Dutch market. These figures are for a single farm participating in the market. As for the prediction tools used, [Holttinen 2005], [Usaola 2007] and [Angarita 2007(1)] use existent prediction tools, while [Fabbri 2005] works with accuracy estimations available in literature. Other studies, such as [Matevosyan 2007] simulate in some way a wind power prediction tool. Even persistence has been used by [Bathurst 2002] for this kind of studies. As for the imbalance prices, their accurate consideration is a difficult task, since they are difficult to forecast and the available data are sometimes not available. [Fabbri 2005] uses the reserve prices in the Spanish market, [Matevosyan 2007] also estimates the imbalance cost from the (already past) reserve prices, and [Holttinen 2005] uses real prices already known. [Usaola 2007] assumes a given amount and makes a sort of sensitivity study, and [Angarita 2007(1)] considers the average imbalance costs.

The most effective way of reducing these losses is to make use of the portfolio effect, or the error reduction when different wind farms join their forecasts to form an only prediction, and therefore a combined bid. [Fabbri 2005] finds that a reduction of between 1 and 2% of maximal revenue could take place, but it has been stated that this effect could render a reduction of a 50% of the total imbalance losses has been reported by [Ceña 2006]. The difference between both estimations could be due to different assumptions in the respective studies.

This amount of losses could be reduced even more. Since the accuracy of prediction tools depends, to some extent, on the time horizon of the prediction, to update the bid in shorter term markets is profitable. If a time horizon between 3 and 7 hours is considered, then the imbalance losses may decrease of 2% of the maximal revenue, approximately, as shown in [Fabbri 2005], [Holttinen 2005], [Angarita 2007(1)]. A shorter term market, as for instance, those of the UK or the intraday French market would provide even better results for the wind farm. Under these conditions, forecasting tools would be almost useless [Bathurst 2002], although it might be useful for uncertainty modeling even under these conditions.

Imbalance losses can be still decreased if the uncertainty of wind power prediction is taken into account and a probabilistic optimization process is followed. [Usaola 2007] finds a reduction in losses of a 3% of the maximal revenue, if such a strategy is used, and [Pinson 2007] obtains a decrease of 5.2% under similar assumptions. [Bathurst 2002] also proposes such a strategy, but the uncertainty considered in his study is too high, due to the fact that he does not use a forecasting tool. None of these studies consider the uncertainty of prices, nor any model the uncertainty of the future imbalance cost, so that different results would be obtained under more realistic assumptions. This issue is presently under study by different research groups.

It must be remarked that an important part of wind farm revenue comes from subsidies which are not linked to the imbalances. The relative importance of imbalance on the total revenue is therefore reduced compared to the real impact they generate. The imbalance cost may be higher if no support mechanisms are considered. Under different conditions, the imbalance costs may have a higher signification for wind power producers. The numerical results of some of the considered papers are summarized in Table 1.

5 Optimal bids.

The problem considered here is the optimization of the bid to the IM, given a position in the DM, taking into account the uncertainties of the involved random variables. The imbalance cost will be taken as a parameter in order to obtain the sensitivities of the revenues with respect to these costs.

Under these conditions, the revenue R_t obtained by a wind farm participating in the electricity market in a Settlement Period t is a random variable that depends on other random variables, namely the power production $P_{g,t}$ and the IM price $\pi_{i,t}$. The aim of the problem is to obtain the bid in the intraday market $P_{i,t}$ that maximimizes R_t. Since the optimization problem is independent for each time t, this subscript will be dropped hereon. Mathematically R can be expressed as

$$R = g(P_g, \pi_i; P_i) \tag{4}$$

Where g is given by equation (1)

The expected revenue will be, then,

$$\overline{R} = E\left[R; P_i\right] = \int_{-\infty}^{\infty} R f_z(R) dR = \int \int_{-\infty}^{\infty} g(P_g, \pi_i; P_i) f_z(P_g, \pi_i) dP_g d\pi_i \tag{5}$$

where $f_z(R)$ is the Probability Density Function (PDF) of the random variable R. If we consider that P_g and π_i are independent random variables [3], then it may be written that

[3] Actually they are dependent if we consider that all wind farms in a market experience the same imbalance and participate in the IM. This dependence is still to be studied.

Table 1: Summary of the results of some of the examined papers.

	Holttinen	Fabbri			Usaola		Angarita		Pinson	
Year	2000	2005			2006		2007		2007	
Wind power (MW)	2000	24.6	301.7	5000	14		14		15	
Imbalance cost (p.u. MP)	0.376	0.485			0.5		0.266		0.215	
Buy	1.27	1.46			1.5		-		1.07	
Sell	0.51	0.49			0.5		-		0.64	
Losses (% max. revenue)	12.11	11.3	9.7	10.8	10.39	7.32	4.5	9	13.1	7.9
Comments	(1)	(2)	(3)	(4)	(6)	(7)	(8)	(9)	(11)	(12)
		(5)					(10)			

(1) *Results for the 13-37 hours forecasts. Results for shorter term are given in the paper.*
(2)(3)(4) *Results for the powers shown above.*
(5) *Results for 48 hours forecasts. Results for shorter term are given in the paper.*
(6) *Best prediction bid in daily market, updated in four intraday markets.*
(7) *Results for the strategic bid, taking into account the uncertainty of wind predictions and the asymmetry of the imbalances' costs.*
(8) *Results with subsidies.*
(9) *Results without subsidies.*
(10) *Results for the Spanish markets only for the daily markets. Results for the UK market and other assumptions are given in the paper. Imbalance costs are considered equal whether under or overprediction.*
(11) *Results for the best prediction bid.*
(12) *Results for the strategic prediction.*

$$f_z(P_g, \pi_i) = f_{P_g}(P_g) f_{\pi_i}(\pi_i)$$

where $f_{P_g}(P_g)$ and $f_{\pi_i}(\pi_i)$ are the PDF of the random variable P_g and π_i).
The optimization problem, then, can be posed as

$$P_{i,opt} = \arg\max_{P_i} E\left[R; P_i\right] \tag{7}$$

Where $P_{i,opt}$ is the optimal position to be taken in the IM. This problem may be solved easily by simple enumeration, an it must be solved for each time t.

6 Dependence of market prices on the wind power submitted.

It is clear that the participation of wind power into the electricity market will have a certain influence on the system price, specially in those markets, such as the intraday markets, with low liquiditye, and where a sizeable amount of power may modify in a significant way the price level. Without considering by the moment the quantification of liquidity, this dependence could be considered in the following way.

Let ΔR be the increase/decrase in revenue of a wind farm that participates in the intraday market in an given period t. The subscript t will be dropped for simplicity,

The mathematical expression of this incremental revenue would be given by 8.

$$\Delta R = \pi_i(P_i - P_d) + IC \tag{8}$$

where

$$IC = \begin{cases} \pi^s(P_g - P_i) & P_g > P_i \\ \pi^b(P_g - P_i) & P_g < P_i \end{cases}$$

where π_i is the intraday market price, P_i is the power bid in the intraday market, P_g is the actual power generated, and P_d is the power bid in the previous daily market. Let us call $\Delta P_i = P_i - P_d$ and $\Delta P_g = P_g - P_i$.

The following assumptions will be considered:

- The liquidityof the market is not considered in the formulation.
- All the wind power participates in the intraday market.
- The prediction error of all wind farms are similar, since them come from similar weather forecasting system.
- All the wind farms will follow the same strategy. Therefore, they will sell if they have more wind than predicted, and it is profitable, and vice versa.

Then one possible modelling of the dependence of intraday price on the power bid is given by 9

$$\pi_i = \pi_i^o + k_i \Delta P_i \tag{9}$$

where π_i^o is the price without any wind power, and k_i a parameter to be determined. It can be seen that π_i decreases if power is bought in the intraday market.

In a similar way, the imbalance price will change because the amount of imbalance in the system will change with the wind power updated in the intraday market. The expression could be given in this way:

$$\begin{array}{ll} \pi^s = \pi_o^s - k_s \Delta P_g & \Delta P_g > 0 \\ \pi^b = \pi_o^b - k_b \Delta P_g & \Delta P_g < 0 \end{array} \tag{10}$$

k_s and k_s are also parameters. π_o^s π_o^b are the average imbalance prices. It may be seen in equation (10) that the sell price decreases if there is more energy declared, and the buy price increases if there is more energy generated.

7 Study Cases.

7.1 Uncertainty of Wind Power Predictions.

7.1.1 Assumptions.

The assumed hypothesis for the study have been the following:

- A pool system has been considered. Wind producers make their bids for a given amount of power at price zero. This means that bids are always accepted.
- The prediction of the prices of the intraday market are perfect.
- Prices in the intraday market do not depend on the amount of wind power bid.
- The subsidies for wind energy are not considered.
- The prediction tool makes new prediction from available data (wind forecasts and real-time production) every hour.
- SIPREÓLICO has been the prediction tool used for performing the prediction. The performance of this program may be considered as representative, as shown in the comparative study [Kariniotakis 2006]. Information about SIPREÓLICO may be found in [Gonzalez 2004].

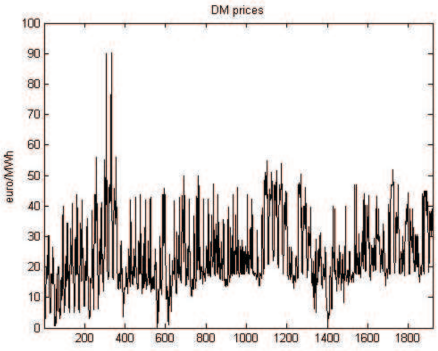

Figure 12: Prices for the period considered.

7.1.2 Data.

The data of wind farm come from the actual production of a wind farm of 14 MW of rated power during three months. The PDF of the wind farm have been obtained from these same production data and predictions performed for this wind farm for these three months. Although the study conditions do not follow the Spanish market rules, the prices of the Spanish market between January and March 2003 have been used for this study. The level of the prices is given in Figure 412. The average price for this period was 23.678 Euro/MWh. The intraday market prices for this period where also used. The average intraday market price used was 22.4791 Euro/MWh.

The study performed includes a comparison between three different assumptions:

1. OPTIMAL: The described method of maximizing the revenues taking into account the uncertainties of the wind power prediction tool.
2. BEST PREDICTION: When the best prediction available is used to modify the bid in the intraday markets.
3. NO INTRADAY: When no updating is produced in the intraday market. Therefore, only one bid per day is produced.

The study has been performed for eighty days. Longer studies, however, do not lead to very different results.

7.1.3 Results and Comments.

For the three assumptions, and for values of $\pi^{sell} = 0.5 \cdot \pi_d$ and $\pi^{buy} = 1.5 \cdot \pi_d$, the revenues, the average errors and the average of the absolute value of the errors along the whole period are presented in Table I. It can be seen there that the most profitable option presents larger errors than the most accurate. Errors and absolute errors for instant t are defined as:

$$\begin{aligned} error_t &= P_{i,t} - P_{g,t} \\ abserror_t &= |P_{i,t} - P_{g,t}| \end{aligned} \tag{11}$$

The values of $\pi^{sell} = 0.5 \cdot \pi_d$ and $\pi^{buy} = 1.5 \cdot \pi_d$ are realistic in the sense that, usually, to buy energy at the last moment is more expensive than to sell it. As these values are too high, this results can be taken as un upper limit.

Figure 13 shows the differences between the three cases. The asterisks (*) show the difference between the OPTIMAL and the BEST PREDICTION assumptions. The diamonds show the difference between the OPTIMAL and the NO INTRADAY assumptions. The dotted line shows the difference between the

	OPTIMAL	BEST PREDICTION	NO INTRADAY
Revenue(Euro)	290112	280500	269046
Error (MW)	-0.3661	0.1412	0.1619
Abolute error (MW)	1.8772	1.4132	2.0198

Table 2: Errors for the different assumptions

Buy/sell prices	1.5/0.5	1/1	0.5/1.5
Optimal	290112	348834	467178
Best prediction	280500	311751	343002
No intraday	269046	313039	357032

Table 3: Revenues for different sell and buy prices (Euro)

BEST PREDICTION and the NO INTRADAY assumptions. It may be observed that the difference, especially between OPTIMAL and BEST PREDICTION is almost always greater than zero. Only in some cases where the predictions had been bad, the results for the BEST PREDICTION were better than the OPTIMAL assumptions.

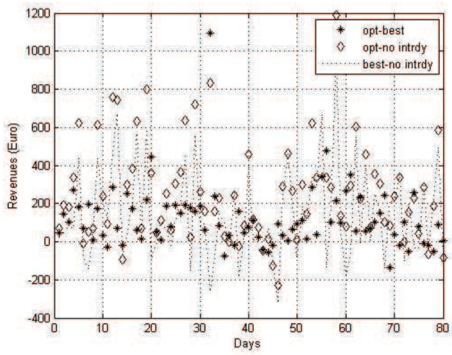

Figure 13: Results for one month of the cases studied.

However, if we check the error between the actual generated power and the different powers, as shown in Table 2, we can see that, even if the errors of the OPTIMAL assumptions are greater, the revenues obtained are higher, and then, the bids provided by the wind farm owners aiming to maximize their revenues are not the most accurate.

From Table 2 we can deduce that the prediction tool has a trend for overprediction, because the errors, defined as in (11) are positive. This is more apparent when the buy and sell prices change from those assumed in the previous results. Although the following discussion is mainly of theoretical interest, it gives a good insight into the properties of the process. The Table 3 gives different results for different values of sell and buy prices. The second column has the same values as the first row of Table 2, for an easier comparison.

We can see in this table that:

1. The revenue when bidding the optimal power is always the highest of the three possible bids.

2. The revenue is higher when the sell price becomes greater. This means that the system tends to overpredict, and then, power must be bought at the last moment most of the times.

3. When the sell price is equal or lower than the buy price, the revenues are larger when no updates are made in the intraday markets. This is also a consequence of the tendency to overpredict, since the error is higher in this last case, as shown in Table 2.

Buy/sell prices	1.5/0.5	1/1	0.5/1.5
Optimal	-0.3661	-0.2423	0.6255
Best prediction		0.1412	
No intraday		0.1619	

Table 4: Average errors for different sell and buy prices(MW).

Buy/sell prices	1.5/0.5	1/1	0.5/1.5
Optimal	1.8722	4.2822	5.5945
Best prediction		1.4132	
No intraday		2.0198	

Table 5: Average absolute errors for different sell and buy prices(MW).

In order to show the bias of the errors and absolute errors, these errors are given in Tables III and IV. In them, the values for the BEST PREDICTION and NO INTRADAY assumptions given in Table I are also included for an easier comparison.

From these tables we may conclude also that:

- The lowest errors are always given by the BEST PREDICTION case. This better performance, however, does not lead to the highest revenues, even compared with the NO INTRADAY option.
- The errors from the OPTIMAL case show that with this strategy, the trend to overpredict of the prediction tool is compensated. When the sell price is higher than the buy price, the bid tends to be higher than the actual generation. The fact that the average error is negative when both prices are equal is another consequence of this trend to overprediction.

The trend with this strategy is to arbitrage between the Intraday Market Price and the imbalance cost. Two examples can be given:

- If the prices of the DM are high (and consequently, the sell price, related to it), and the IM prices low, the trivial strategy is to buy energy in the IM and to sell at the spill price in real time market.
- If the prices of the DM are low (and hence the buy price), and the IM prices high, the strategy in this case is to sell as much as possible in the intraday market and to pay the buy price for the energy not furnished.

7.2 Uncertainty of Wind Power Predictions and Intraday Prices.

A simulation study will be carried on under the same assumptions of Section 7.1.1 and with the same data. In this case, however, the assumption of perfect knowledge of the price will not be taken, and, instead, the error and uncertainty of an intraday price prediction tool will be simulated. The price in the intraday market is still considered independent of the wind farm position.

7.2.1 Price predictions.

A price prediction tool will be simulated here, and the results of the next section will be evaluated according to the accuracy of this tool. The simulation will consist in adding a term to the known intraday price, and to model the uncertainty as a random variable with a given variance. The bias of the prediction tool will be assumed to be zero. The base case values will be taken similar to ordinary prediction tools. Hence, the predicted price will be obtained as

$$\widehat{\pi}_{i,t} = \overline{\pi}_{i,t} + \varepsilon_i \tag{12}$$

where $\varepsilon_{i,t}$ is a random normal variable $N(0, \sigma_\varepsilon)$ and $\overline{\pi}_{i,t}$ is the average IM price in the period studied. The standard deviation of the random variable $\pi_{i,t}$, will be also σ_ε

		High	Normal	Null
Normal	(MW)	121	224	304.1
	(%)	0.96	1.65	0.23
Gamma	(MW)	37.26	64.77	38.41
	(%)	0.29	0.48	0.529
Volume	(MW)	12595	13609	13190

Table 6: Differences between power bid with and without considering the uncertainty of IM price prediction

	Revenues (kEuro)	
σ_ε	1.1/0.8	1.5/0.5
0	327.90	291.30
0.2	327.89	291.28
0.45	327.74	291.2
1	327.08	291.02
2	324.93	289.89
5	319.58	284.97
Best	304.76	281.40

Table 7: Revenues with different accuracies of the price prediction tool.

7.2.2 Results and Comments.

Importance of the uncertainty of price predictions. Different hypothesis have been assumed in order to quantify the importance of modelling the uncertainty of the price prediction. Three different imbalance costs have been assumed:

- High costs: 1.5/0.5 Daily Market Price (DMP)
- Normal costs: 1.1/0.8 DMP
- Null costs: 1/1 DMP

In this case, the standard deviation of the price error and the price uncertainty is $\sigma_\varepsilon = 0.45$.

Two different PDF of the price uncertainty have been considered: Normal (symmetric PDF) and Gamma (asymmetric PDF). The parameter that will be compared is the value

$$\varepsilon_{of} = \sum_{t=0}^{T} |\pi_{i,t}^{nounc} - \pi_{i,t}^{unc}| \tag{13}$$

where $\pi_{i,t}^{nounc}$ is the power bid when the uncertainty is not considered, while $\pi_{i,t}^{unc}$ is the power bid when the uncertainty is considered.

The results are presented in Table 6. The last row is the total power negociated in the IM during the whole period, in MW.

From Table 6 it can be seen that the importance of considering the uncertainty of the intraday price prediction is very small, although it increases when the imbalance cost decreases. In these conditions there are more possibilities of arbitraging in the market.

Importance of price prediction accuracy. Another interesting study is to evaluate the revenues with different accuracies of the price prediction tool. The results will be presented for the case of normal (1.1/0.8) and high (1.5/0.5)imbalance costs, and no uncertainty of the prediction tool will be considered. The results are given in Table 7. The parameter is the variance of the error σ_ε. The revenues obtained if the best prediction is bid, are given in the last row of the table.

The influence of the prediction accuracy does not seem to be very high, although it is significative for very inaccurate predictions.

Random imbalance price. In order to simulate somehow the random behaviour of the imbalance prices, a random imbalance price generator, with a given mean has been made for the two prices system imbalance price (see section 2.1). The probability of having a positive system imbalance (overgeneration) is 50%, and the imbalance price is modelled as a Beta variable of a given mean and variance. This random variable is independent of the imbalance of the wind farm.

The results are identical to those obtained with the mean imbalance price.

8 Conclusion.

From the simulations run, and under the assumed hypothesis, the following preliminar conclussions could be drawn.

- The losses for a wind power producer are significant, and about a 10% of the maximum expected revenues. This figure can be reduced by updating bids, joining bids among several wind farms, and strategic bidding.
- The strategic biddings uses the uncertainty of the wind power predictions, and arbitrages between expected prices of intraday markets and expected imbalance costs.
- The modelling of the uncertainty of the price prediction does not affect the final bid, for normal values of imbalance costs.
- The accuracy of the price prediction tool has a moderate impact on the revenues obtained with strategic bidding.
- The impact of large amounts of wind power participating at the same time in the intraday markets, and with similar trends, will have an impact on the results of these markets, still to evaluate.

9 Future studies.

The studies developed here may be continued in the following directions:

- Modelling of imbalance price, trying to model somehow this uncertainty.
- Quantification of the influence of wind updates and participation in the daily and intraday markets on the prices of these markets, taking into account their liquidity.
- Risk analysis of the bidding strategy, in order to limit the amount of maximum losses to be produced due to big prediction errors.

A Statistical distributions.

A.1 Beta distribution.

The analytical expression of beta probability density function is

$$f(x; a, b) = \frac{1}{B(a, b)} x^{a-1}(1 - x)^{b-1}$$

Where $B(a, b)$ is the beta function, and a and b are parameters related to the mean, η , and the variance, σ^2, in the following way:

$$\eta = \frac{a}{a+b} \qquad \sigma^2 = \frac{ab}{(a+b)^2(a+b+1)}$$

The beta distribution has been represented in Figure 14.

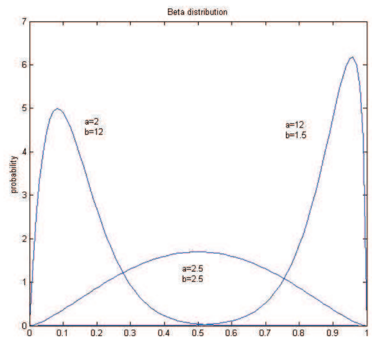

Figure 14: Beta distribution for different values of parameters a and b.

A.2 Binomial distribution.

Binomial distribution is the discrete probability distribution of the number of successes in a sequence of n independent yes/no experiments, each of which yields success with probability p.

In general, if the random variable K follows the binomial distribution with parameters n and p, we write $K \sim B(n, p)$. The probability of getting exactly k successes is given by the probability mass function:

$$f(k; n, p) = \binom{n}{k} p^k(1 - p)^{n-k}$$

The mean of this distribution is $\eta = np$, and the variance is $\sigma^2 = np(1 - p)$.

A.3 Exponential distribution.

In probability theory and statistics, the exponential distributions are a class of continuous probability distributions. An exponential distribution arises naturally when modeling the time between independent events that happen at a constant average rate.

The probability density function (pdf) of an exponential distribution has the form:

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x} & , \quad x \geq 0 \\ 0 & , \quad x < 0 \end{cases}$$

The mean of this distribution is $\eta = \frac{1}{\lambda}$, and the variance is $\sigma^2 = \frac{1}{\lambda^2}$.

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